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Algebra 1

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Quiz #1

Problems:

1. (1pt) Give an example of two matrices $A, B \in GL_2(\mathbb{R})$, such that their product is not commutative, that means that $AB \neq BA$.

   Solution: *Take* $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ *and* $B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$,

$$AB = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

2. (1pt) List the elements of the group U_{15} of the units of the set $\mathbb{Z}/15\mathbb{Z}$.

   Solution:

$$U_{15} = \{k \in \mathbb{Z}/15\mathbb{Z} | gcd(k, 15) = 1\} = \{1, 2, 4, 7, 8, 11, 13, 14\}$$

3. (1 pt) Show that if G is a group with only 2 elements then there exists an isomorphism between G and $\mathbb{Z}/2\mathbb{Z}$.
   Solution: *Let G be a group with two element, $G = \{e, a\}$, then we define the morphism $\mathbb{Z}/2\mathbb{Z} \to G$ sending $[0]$ to e and $[1]$ to $[a]$ you can check easily it is an isomorphism of groups.*

4. (2pt) Let G be a group and let H be a subgroup of G. Let x be an element of G. Define $xHx^{-1} := \{xhx^{-1} | h \in H\}$. Prove that xHx^{-1} is a subgroup of G. (Be precise and do not forget steps!)
   Solution: *xHx^{-1} is a subset of G since G is a group and $H \subset G$.*
   $e \in xHx^{-1}$ indeed $e = xex^{-1}$ and $e \in H$, since H is a subgroup.
   For any xhx^{-1} and $xgx^{-1} \in xHx^{-1}$, we have

$$xhx^{-1}(xg^{-1}x^{-1})^{-1} = xhx^{-1}xgx^{-1} = xhg^{-1}x^{-1} \in xHx^{-1}$$

   Proving that xHx^{-1} is a group.